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Sigmoid Based PID Controller Implementation for Rotor Control

Abdullah Ateş, Barış Baykant Alagöz, Celaleddin Yeroğlu, Hafız Alisoy

***Abstract*— This paper presents a sigmoid based variable coefficient PID (SBVC-PID) controller design for Twin Rotor MIMO System (TRMS). The proposed SBVC-PID controller dynamically changes controller coefficients according to a modified sigmoid function of the error signal. The modified sigmoid function is used to limit variability of PID controller coefficients in a predefined range. In the proposed method, each parameters of PID, namely k_p, k_i and k_d, alter between predefined upper and lower bounds. A modified sigmoid function adjusted by a transition coefficient is used to alter each of the PID parameters between these bound limits. The variable coefficients of SBVC-PID maintain a hypercube in k_p, k_i and k_d parameter space satisfying robust stability of the system. Well-known Kharitonov polynomials are used to ensure that the SBVC-PID coefficient alteration takes place in the robust stability intervals. Due to dynamically change of PID coefficients depending on magnitude of error signal, the control performance can be improved compared to conventional PID control. Performance of SBVC-PID controller is demonstrated via theoretical examples and TRMS rotor control simulations.**

I. INTRODUCTION

The Proportional-Derivative (PD), the Proportional-Integral (PI) and the Proportional-Integral-Derivative (PID) controllers have found wide application area. PID control structures were firstly proposed by Minorsky in 1922 [1] and Callender in 1936 [2]. PID controllers have been commonly preferred in industrial control applications due to its simplicity in design and implementations. These advantages make the PID control techniques highly preferable for process control applications. Nowadays, studies for the enhancement of PID controller performance are very prominent for control practice. Stability and tuning problems of PID controllers for performance enhancement were addressed in many works; Ziegler-Nichols tuning [3-5] and Kitamori's PID design methods was proposed for process control [6]. Later, auto-tuning of PID controllers was demonstrated by using heuristic optimization methods such as stochastic multi-parameters divergence optimization (SMDO) algorithm [7], an artificial bee colony, harmony search and bee algorithms [8], tabu search algorithm [9]. There are also many efforts to modify classical PID structure to obtain better control performance such as variable structure PID [10], fractional-order PID [11].

A. Ateş and C. Yeroğlu are with Computer Engineering Department, Inonu University, Malatya, TURKEY (corresponding author to provide phone: 00904223774883,ext:152; e-mail: abdullah.ates@inonu.edu.tr).

B. B. Alagoz was with Electrical and Electronics Engineering Department, Inonu University, Malatya, TURKEY (e-mail: baykant.alagoz@inonu.edu.tr).

H. Alisoy is with Electronics and Telecommunication Eng., Namik Kemal University, Tekirdag, TURKEY (e-mail: halis@inonu.edu.tr).

Variable control structure has been studied almost for twenty years [12-17]. Variable coefficient controllers are reported to be effective for the control application containing parameter perturbation with lower and upper bounds, nonlinearities, external disturbances [13]. Variable control structure is extensively used in sliding mode control [18-21]. In the literature, variable structure PID (VS-PID) term is mainly used for switching control. PID coefficients were altered in order to improve controller performance depending on switching strategies according to state vector and shifting control trajectory in a partitioned state space [12]. Variable control structures provide instant parametric perturbations and it deals with nonlinearity in control systems. Especially, VS-PID controllers have been utilized in the level control processes [12], robot manipulators [13], particularly, for the control of high disturbance and nonlinear systems.

In general, PID tuning methods were developed to find optimal controller parameters for linear time invariant plants. Generally, performance of linear controllers depends to the optimal parameters and these parameters have fixed value. However, it can be possible to generate more appropriate control responses, when the controller parameters are changed depending on nonlinear dynamics of control system. In this manner, we aimed to suggest a straightforward method to turn a conventional PID into a variable structure PID. For this proposes, we perturb each PID coefficients in a predefined ranges depending on magnitude of error signal by using a modified version of sigmoid function. The modified sigmoid function allows nonlinearly compression of controller coefficients between the upper and lower bounds. Transition between bounds of parameters takes place asymptotically and continuously by using exponential function characteristics. One of the advantages of using variable PID coefficients depending on error signal is that the error signal well reflects dynamics of control system and one can establish different PID response for different error states to improve controller performance. SBVC-PID controller fundamentally gives us an opportunity to regenerate PID controller according to error states of control systems.

We present two theoretical examples to compare performance of SBVC-PID with standard tuned PID controller. Then, we tested SBVC-PID in TRMS simulations. TRMS experimental system is developed to simulate flight control of twin rotor system and TRMS control presents a complicated control problem due to aerodynamics, nonlinearity, and effect of environmental conditions. The proposed variable control strategy, which is based on alteration of SBVC-PID controller parameters, changes the optimal PID to smooth PID to enhance control performance with a faster step response and lower maximum overshoot [7]. Since the stability of interval systems have been investigating extensively in control applications [22-24], the stability problem of SBVC-PID control system is discussed on the bases of robust stability of interval systems in the proposed method. In order to ensure the stability of system with SBVC-PID, we also carried out stability analyses by using Kharitonov polynomials.

Following section provides a methodology for SBVC-PID with the sigmoid function. Section 3 presents simulation examples to show control performance of SBVC-PID. Last section gives some conclusions.

II. Sigmoid Based Variable Coefficient PID Control

The sigmoid function, also called the sigmoidal curve or logistic function is a continuous form of the ramp function [25]. It is famously known as an activation function of artificial neural networks. Sigmoid function is mainly used to limit system output and it is given as follows:

$$f(x)=\frac{1}{1+e^{-\alpha x}} \quad (1)$$

where, the parameter α is a transition coefficient that is used to adjust sharpness of transition between lower and upper bounds as shown in Fig. 1. The sigmoid function compresses the value of x between zero and one, asymptotically. This property makes the sigmoid function be a good limiter that is continuous and differentiable. For this reason, we preferred sigmoid function as limiter function of PID coefficients for SBVC-PID controller. This function was used to model power limited systems in power system simulations [26]. Use of modified sigmoid function as a limiter function in control system was demonstrated for the peak power limitation of DC/DC converters [27]. In the current study, we used the modified sigmoid function to alter PID coefficients continuously within the predefined bounds. VS-PID controllers were generally designed by sliding surface [13], fuzzy controllers [28]. In this study, we aim to change PID coefficients between two states to obtain desired response depending on control dynamics.

For the proposed SBVC-PID controller, PID coefficients are changed by using (2), which is a modified version of (1),

$$f(x)=x_{low}-\left|\frac{x_{high}-x_{low}}{1+e^{-\alpha|e(t)|}}\right| \quad (2)$$

where x_{low} and x_{high} are lower and upper bounds of the coefficients, and $e(t)$ is an error function as shown in Fig 2.

When input signal is applied to variable structure control system, the system should be stable while altering controller coefficients according to control error signal. Values of the x_{low} and x_{high} can be obtained $x_{high}=x_o+\Delta x$ and $x_{low}=x_o-\Delta x$ where x_o is a nominal value of the coefficient and Δx is the amount of deviation. When this formulation is applied for all PID coefficients, PID coefficients can be written as follows,

$$k_{pv}=k_{plow}-\left|\frac{k_{phigh}-k_{plow}}{1+e^{-\alpha|e(t)|}}\right| \quad (3)$$

$$k_{iv}=k_{ilow}-\left|\frac{k_{ihigh}-k_{ilow}}{1+e^{-\alpha|e(t)|}}\right| \quad (4)$$

$$k_{dv}=k_{dlow}-\left|\frac{k_{dhigh}-k_{dlow}}{1+e^{-\alpha|e(t)|}}\right| \quad (5)$$

Using (3), (4) and (5), transfer function of a SBVC-PID controller can be written in general form as follows:

$$PID=k_{pv}+\frac{k_{iv}}{s}+k_{dv}s \quad (6)$$

Fig. 2 shows block diagram of closed loop SBVC-PID control system. VS-PID term is mainly used for switching control, which switches PID coefficients to improve controller performance. The main advantage of the proposed method is that the modified sigmoid function provides continuously altering of PID coefficients. Change of PID coefficients takes place in time–continuously and this prevents interruption and jumps of control actions.

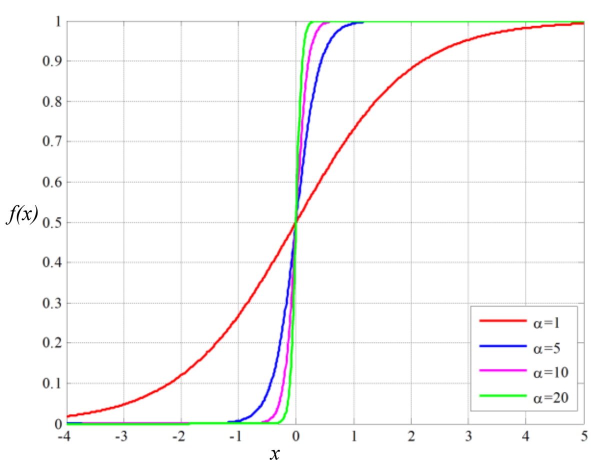


Figure 1. Sigmoid Functions for different α values.

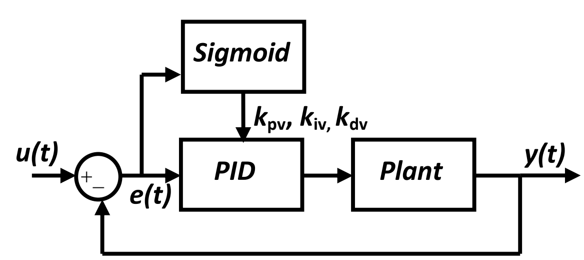


Figure 2. Closed loop SBVC-PID control systems

III. Simulation Result

A. Illustrative Examples

In this section, we present simulation examples previously studied in [8, 29] to show advantage of SBVC-PID.

Example 1:

Fig. 3 shows step response of SBVC-PID and optimized PID in [8, 29] for the plant $\frac{1}{s^2+2s+1}e^{-0.5s}$. The coefficients of SBVC-PID used in the simulation is given in Table 1. The simulation results show that the SBVC-PID controller can provide a lower maximum overshoot for almost the same time settling time. This improvement is a result of the evolution of SBVC-PID towards a slower controller, where the magnitude of error decreases.

Example 2:

Fig. 4 compares the step response of SBVC-PID and the results in [8, 29] for a third-order plant, $\frac{4.228}{s^3+2.14s^2+9.276s+4.228}$ [8, 29]. The simulation results confirm that SBVC-PID can provide lower maximum overshoot compared to other designs methods for the almost the same settling time. Table 2 gives SBVC-PID coefficients used in the simulation.

B. TRMS Simulations:

The TRMS is widely used for flight control simulations. TRMS is a two-rotor system that maneuvers by vertical and horizontal rotors [30-32]. TRMS experimental setup is presented in Fig 5.

TRMS control problem introduces a highly nonlinear Multi-Input-Multi-Output (MIMO) problem [33]. In order to simplify the mechanical design of the fight system, the blades of the rotors have a fixed angle of attack, and the control is achieved by controlling the speeds of the rotors [34]. Performance of SBVC-PID controller was tested for control of TRMS. Many works in the literature were addressed TRMS control problem: Jih-Gau-Juang et al. developed several classical control schemes and intelligent control schemes of an experimental propeller setup for TRMS system in [35]. Alagoz et al. used SMDO algorithm for the auto-tuning of PID controller according to a fractional order reference model to obtain a smooth step response [7]. Then, Yeroglu et al. applied SMDO method for auto-tuning of fractional order PID controllers for TRMS in [11].

In order to perform stability analyses of SBVC-PID controller used for TRMS system, we obtained a dynamical model of TRMS system. These dynamical models were used to determine the lower and upper bounds of SBVC-PID, where the TRMS system stays robustly stable.

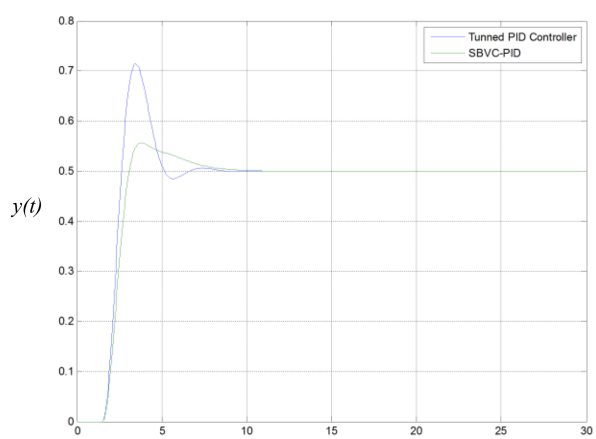
Dynamic modeling of TRMS is performed as follows: Multi-sinusoidal signal composed of the sum of four harmonics with 2.199, 0.42, 0.94 and 1.5 *Hz*. was applied to TRMS experimental setup during 100 seconds. Then, the responses of rotors were recorded by data capturing. These data, containing system response for the multi-sinusoidal input, were used to derive system model by using MATLAB system identification toolbox. Second order OE (Output-error) linear parametric model identification structure was used for obtaining TRMS dynamical mathematical model of rotors.

TABLE I. Lower and Upper bounds of SBVC-PID obtained by proposed method for the plant in Example 1.

	Low	High
k_{pv}	2.308	3.308
k_{iv}	1.2121	2.2121
k_{dv}	0.6512	1.6512

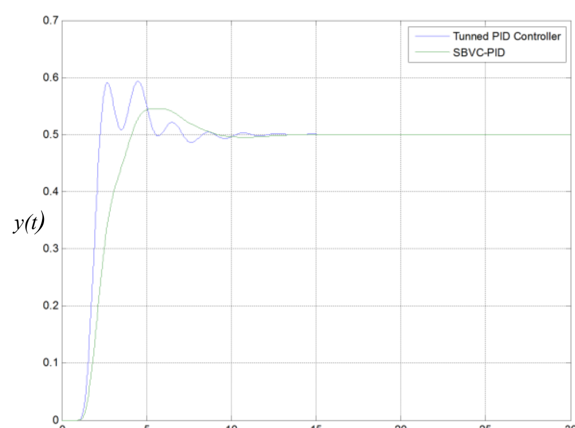
TABLE II. Lower and Upper bounds of SBVC-PID obtained by proposed method for the plant in Example 2.

	Low	High
k_{pv}	1.19	3.19
k_{iv}	0.1262	3.1262
k_{dv}	0.265	0.865



Time (sec.)

Figure 3. Step response of (8) for classical PID and SBVC-PID (for $\alpha=1$).



Time (sec.)

Figure 4. Step response of classical PID and SBVC-PID (for $\alpha=10$).

Main rotor mathematical models are obtained as follows,

$$G(s)=\frac{0.06725s+1.359}{s^2+0.7906s+3.666} \quad (7)$$

Fig. 6 shows the Bode plots of the generated mathematical models. As seen from the Bode plots, the plant model is stable.

The upper boundaries of SBVC-PID coefficients were obtained by SMDO algorithm in [7], the lower boundaries of SBVC-PID coefficients were determined according to optimal PID of TRMS in [30]. The upper boundary of coefficients provides altering of SBVC-PID response to a smooth response near to set-points, where error magnitude is low. This smooth response was previously optimized according to a fractional-order reference model to rotor control [7]. At lower boundary of coefficients, SBVC-PID provides a faster response for high error magnitudes, particularly, before converging to the set-point. Table 3 shows the lower and upper bounds of SBVC-PID coefficients.

V. L. Kharitonov [23, 24] states that the interval polynomial family is Hurwitz stable if and only if four special, well defined polynomials are Hurwitz stable. In order to confirm stability of the closed loop SBVC-PID system at two bounds of PID coefficients, we employed Kharitonov theorem.

Let, represented plant functions be in the form of $\frac{as+b}{s^2+cs+d}$, where a,b,c,d are coefficients of the dynamical model of TRMS respectively. Characteristic equations of a closed loop systems can be obtained as:

$$\Delta(s)=\left[k_{ib}\right]+\left[k_{pb}+k_{ia}+d\right]s +\left[k_{pa}+k_{db}+c\right]s^2+\left[k_{da}+1\right]s^3 \quad (8)$$

Interval characteristic polynomial of main rotor SBVC-PID control system for dynamic model given by (8) can be obtained by using lower and upper boundary values of SBVC-PID controller in Table 3, as follows:

$$\Delta(s)=\left[1.6725 \quad 2.1721\right]+\left[14.71 \quad 24.99\right]s +\left[10.53 \quad 14.214\right]s^2+\left[1.45 \quad 10.88\right]s^3 \quad (9)$$

Considering (9), four Kharitonov polynomials can be written for the main rotor control system as follows;

$$\begin{aligned} k_1 &= 1.45+10.53s+24.99s^2+2.17221s^3 \\ k_2 &= 10.88+14.214s+14.71s^2+1.6725s^3 \\ k_3 &= 1.45+14.214s+24.99s^2+1.6725s^3 \\ k_4 &= 10.88+10.53s+14.71s^2+2.1721s^3 \end{aligned} \quad (10)$$

Four Kharitonov polynomials are stable according to Routh Hurwitz criterion (See Appendix1 for stability analyses of Kharitonov polynomials). For this reasons, one can state that main rotor SBVC-PID control system is robustly stable in between upper and lower boundary of SBVC-PID coefficients.

Simulations Result

In the simulations, we used SBVC-PID controller designs shown in Table 3 for the main rotor. The upper boundaries of SBVC-PID controller coefficients in Table 3 was obtained by SMDO algorithm [7] and this boundary provides smooth step response with a low overshoot in settling. The lower boundary of SBVC-PID coefficients in Table 3 was taken the optimal PID controller design suggested by producer of TRMS experimental setup (Feedback Instruments Inc.) [30]. Previous section validates the stability of SBVC-PID control system in the variability ranges of controller coefficients.

TABLE III. LOWER AND UPPER BOUNDS OF SBVC-PID OBTAINED BY PROPOSED METHOD FOR TRMS MODEL

	Low	High
k_{pv}	5	7.37
k_{iv}	8	1.07
k_{dv}	10	17.43

Figure 5. TRMS (Twin Rotor MIMO System)

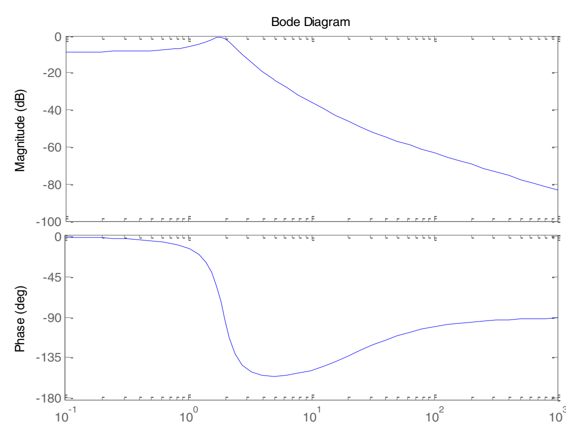


Figure 6. The Bode plots of TRMS main rotor model.

Fig. 7.a compares step responses of optimal PID controller of TRMS [30], smooth PID controller of TRMS [7] and the proposed SBVC-PID controller transforming from the optimal PID controller to smooth controller with respect to error magnitude. As shown in Fig. 7a, SBVC-PID controller presents a better response than the optimal PID controller and the smooth PID controller. However, it provides a lower overshoot and settling time than the optimal PID controller. Transformation character of SBVC-PID controller from the optimal PID controller to smooth controller can be changed by α parameters as illustrated in Fig. 7.b. High value of α results in fast transition between lower and upper coefficients and behaves as if a switch. Low values of α cause a slower change between bounds of coefficients as shown in Fig. 1. The parameter α can be adjusted heuristically to obtain a satisfactory performance for the control application. A noteworthy advantage of SBVC-PID controller in our simulation results is that it can reduces ripples and overshoots compared to conventional PID structure.

IV. Conclusion

This paper introduces SBVC-PID controller design and demonstrates its application in the TRMS rotor control. The SBVC-PID controller implements a dynamically varying PID response depending on error state of the control system and this can provide more appropriate step response compared to conventional PID structures with constant coefficients.

The proposed SBVC-PID method continuously alters PID coefficients between predefined bounds of controller coefficients according to the magnitude of error signal. It uses a modified sigmoid function to alter PID coefficients, which simplifies VS-PID design compared to other VS-PID controller design strategies [13]. In SBVC-PID controller design process for TRMS system, we preferred a VS-PID controller, which transforms from a fast controller to slow one as error magnitude decreases. This provides a faster controller response with a lower overshoot, in which the conventional PID structure with constant coefficients may not perform.

Simulation results clearly show the performance improvements obtained by SBVC-PID controller and encourage us to employ it for the rotor control problem.

Design process includes two basic tasks: (i) Determination of bounds of PID coefficients for desired controller responses and (ii) the validation of robust stability of the control system in the variability ranges of PID coefficients. In TRMS design example, the optimal PID [30] for fast response at high error magnitudes and the smooth PID [7] for slow response at high error magnitudes were considered as the bounds of SBVC-PID coefficients. Robust stability of the SBVC-PID control system was confirmed by Kharitonov polynomials.

As a consequence, SBVC-PID enhances PID controller performance and contributes to implementation of advanced PID controller structures for control engineering practice.

Appendix

Stability Analyses of Kharitonov polynomials;

For k_1 polynomial; $k_1 = 1.45 + 10.53s + 24.99s^2 + 2.17221s^3$

All roots in Table 4 are in left half plane and therefore k_1 polynomial is stable.

For k_2 polynomial;

$k_2 = 10.88 + 14.214s + 14.71s^2 + 1.6725s^3$

All roots in Table 5 are in left half plane and therefore k_2 polynomial is stable.

For k_3 polynomial; $k_3 = 1.45 + 14.214s + 24.99s^2 + 1.6725s^3$

All roots in Table 6 are in left half plane and therefore k_3 polynomial is stable.

For k_4 polynomial; $k_4 = 10.88 + 10.53s + 14.71s^2 + 2.1721s^3$

All roots in Table 7 are in left half plane and therefore k_4 polynomial is stable.

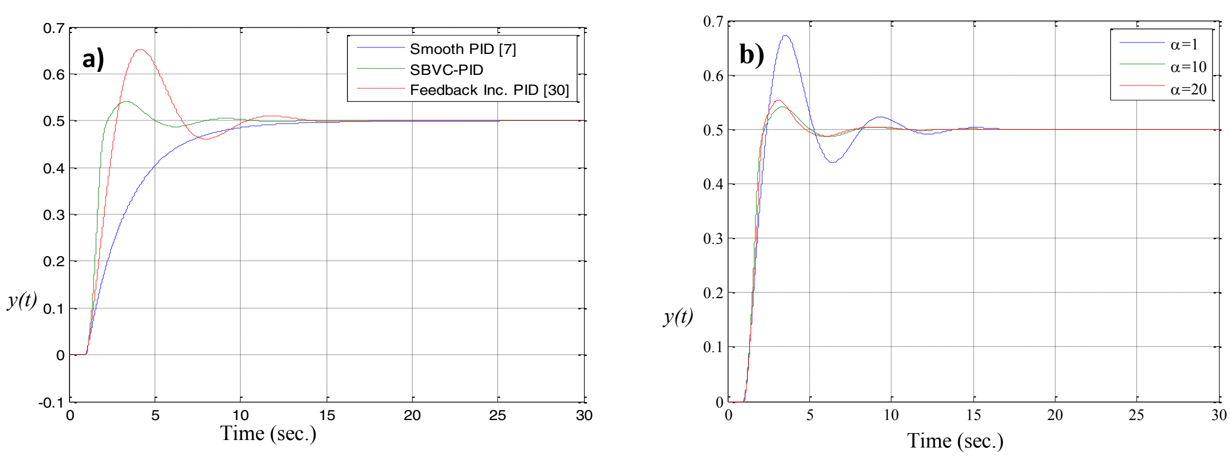


Figure7. (a) Step response of the TRMS main rotor ($\alpha = 10$), (b) Comparison of SBVC-PID controller step response for the different α values

TABLE IV. ROOTS OF k_1

-3.5859+i1.9305
-3.5859-i1.9305
-0.0503

TABLE V. ROOTS OF k_2

-0.6077+i0.9303
-0.6077-i0.9303
-0.1280

TABLE VI. ROOTS OF k_3

-7.5362
-2.1969
-0.0697

TABLE VII. ROOTS OF k_4

-0.4158+i1.0399
-0.4158-i1.0399
-0.1637

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